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LETTER TO THE EDITOR

On the construction of the U matrix from Dirac brackets in QCD

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Abstract. We apply the general procedure developed by Kiefer and Rothe in order to obtain the U matrix in QCD from Dirac brackets. The Coulomb interaction Hamiltonian happens to be that of Christ and Lee.

A general procedure for the construction of the U matrix from Dirac brackets has been obtained by Kiefer and Roethe (KR) [1]. There, as an example, the U matrix for QED was obtained in the temporal and Coulomb gauges. This letter is the result of the possibility of applying this procedure to QCD.

For the application of the Dirac bracket formalism (DBQP) one has as a necessary condition the non-vanishing of the Faddeev–Popov determinant ($\det Q \neq 0$), as is well known [2]. However, it is well established that in compactified QCD this condition is never verified, i.e. it is impossible to find a set of gauge conditions that satisfies $\det Q \neq 0$ [3]. Nevertheless, the Coulomb gauge has been used in QCD via Dirac brackets [4], ignoring the above-mentioned difficulties.

As a first approach in the construction of the U matrix from Dirac brackets in QCD we will also ignore these difficulties. Working in the Coulomb gauge and following KR, we obtain, in analogy to (2.8) of [1],

$$[H_{\rm in}^{(0)}(t), \psi_{\rm in}(x)] = -{\rm i}\gamma^0(\gamma^k\partial^k - {\rm i}m)\psi_{\rm in}(x) + \tfrac{1}{2}\int {\rm d}^3z[\pi_{\rm in}^2(\overset{i,a}{z}), \psi_{\rm in}(x)]. \tag{1}$$

The additional trouble in this case comes from the tentative way of transforming the expression $\frac{1}{2}\int {\rm d}^3z[\pi_{\rm in}^2(\overset{i,a}{z}), \psi_{\rm in}(x)]$ into $[H_{\rm I}^{(2)}(\tau), \psi_{\rm in}(x)]$ because there exists no analytical expression for the QCD propagator $K^{a,b}(z, x)$ at our disposal. However, by the use of the power series expansion in g [5] for this propagator:

$$K^{a,b}(\boldsymbol{x}, \boldsymbol{z}) = -\frac{\delta_{ab}}{4\pi|\boldsymbol{x}-\boldsymbol{z}|} + g\int {\rm d}^3y \frac{1}{4\pi|\boldsymbol{x}-\boldsymbol{z}|}\varepsilon_{abc}A_c^i(y)\frac{\partial}{\partial y^i}\frac{1}{4\pi|\boldsymbol{y}-\boldsymbol{z}|} + \cdots \tag{2}$$

we find that

$$\tfrac{1}{2}\int {\rm d}^3z[\pi_{\rm in}^2(\overset{i,a}{z}), \psi_{\rm in}(x)] = [H_{\rm I}^{(2)}(\tau), \psi_{\rm in}(x)] \tag{3}$$

where $H_{\rm I}^{(2)}(\tau)$ is given by [6]

$$H_{\rm I}^{(2)}(t)=\tfrac{1}{2}g^2\int {\rm d}^3x\,{\rm d}^3y\,D_{\rm IN}^{0,a}(x)G_{\rm IN}^{ab}(x,y)D_{\rm IN}^{0,b}(y)$$

$$+\tfrac{1}{2}g^2\int {\rm d}^3x\,{\rm d}^3y\,{\rm d}^3v\,\varepsilon_{a\beta\gamma}A_{\rm IN}^{i,\beta}(x)\partial_x^i\nabla_{xv}^{-2}\partial_v^k\Pi_{\rm IN}^{k,\gamma}(v)$$

$$\times G_{\rm IN}^{ab}(x,y)D_{\rm IN}^{0,b}(y)+\tfrac{1}{2}g^2\int {\rm d}^3x\,{\rm d}^3y\,{\rm d}^3w\,D_{\rm IN}^{0,a}(x)$$

$$\times G_{\rm IN}^{ab}(x,y)\varepsilon_{bef}A_{\rm IN}^{j,e}(y)\partial_y^j\nabla_{yw}^{-2}\partial_w^l\Pi_{\rm IN}^{l,f}(w)$$

$$+\tfrac{1}{2}g^2\int {\rm d}^3x\,{\rm d}^3y\,{\rm d}^3v\,{\rm d}^3w\,\varepsilon_{a\beta\gamma}A_{\rm IN}^{i,\beta}(x)\partial_x^i\nabla_{xv}^{-2}\partial_v^k$$

$$\times\Pi_{\rm IN}^{k,\gamma}(v)G_{\rm IN}^{ab}(x,y)\varepsilon_{bef}A_{\rm IN}^{j,e}(y)\partial_y^j\nabla_{yw}^{-2}\partial_w^l\Pi_{\rm IN}^{l,f}(w) \qquad (4)$$

with

$$D^{0,a}(z)=\varepsilon_{abc}\pi_{j{\rm in}}^{c}(z)A_{j{\rm in}}^{b}(z)+\tfrac{1}{2}\pi_{\psi{\rm in}}(z)\tau^a\psi_{\rm in}(z)$$

and

$$G^{a,b}(x,y)=\int {\rm d}^3u\,K_{\rm IN}^{a,d}(x,u)\nabla_u^2K_{\rm IN}^{d,b}(u,y).$$

It is important to keep the operator ordering given in the above Hamiltonian. Note that symmetrised and antisymmetrised multiplications are called for in Bose and Fermi terms, respectively.

To obtain the U matrix one follows the same steps of KR from (2.13) to (2.17). In this way we get

$$\mathrm{i}\,{\rm d}U/{\rm d}t=H_{\rm I}U \qquad (5)$$

where

$$H_{\rm I}=H_{\rm I}^{(1)}+H_{\rm I}^{(2)} \qquad (6)$$

with $H_{\rm I}^{(2)}$ given by (4) and

$$H_{\rm I}^{(1)}=-\tfrac{1}{2}\mathrm{i}g\int {\rm d}^3z\,\pi_{\psi{\rm in}}(z)\gamma^0\gamma^kA_{\rm in}^{k,a}(z)\tau^a\psi_{\rm in}(z). \qquad (7)$$

Therefore, we conclude that the KR procedure can be applied to QCD. As a by-product, we note that the Coulomb interaction Hamiltonian [4], deduced earlier by Christ and Lee [7], is here reobtained following a completely independent procedure.

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